Prior Learning Topics

As noted in the previous section on prior learning, it is expected that all students have extensive previous mathematical experiences, but these will vary. It is expected that mathematical studies SL students will be familiar with the following topics before they take the examinations, because questions assume knowledge of them. Teachers must therefore ensure that any topics listed here that are unknown to their students at the start of the course are included at an early stage. They should also take into account the existing mathematical knowledge of their students to design an appropriate course of study for mathematical studies SL.

Students must be familiar with SI (*Système International*) units of length, mass and time, and their derived units.

The reference given in the left-hand column is to the topic in the syllabus content; for example, **1.0** refers to the prior learning for Topic 1—Number and algebra.

Learning how to use the graphic display calculator (GDC) effectively will be an integral part of the course, not a separate topic. Time has been allowed in each topic of the syllabus to do this

	Content	Further guidance
1.0	Basic use of the four operations of arithmetic, using integers, decimals and fractions, including order of operations.	*Examples*: $2(3+4\times7)=62$; $2\times3+4\times7=34$.
	Prime numbers, factors and multiples.	
	Simple applications of ratio, percentage and proportion.	
	Basic manipulation of simple algebraic expressions, including factorization and expansion.	*Examples*: $ab+ac=a(b+c)$; $(x+1)(x+2)=x^2+3x+2$.
	Rearranging formulae.	*Example*: $A=\frac{1}{2}bh \Rightarrow h=\frac{2A}{b}$.
	Evaluating expressions by substitution.	*Example*: If $x=-3$ then $x^2-2x+3=(-3)^2-2(-3)+3=18$.
	Solving linear equations in one variable.	*Examples*: $3(x+6)-4(x-1)=0$; $\frac{6x}{5}+4=7$.
	Solving systems of linear equations in two variables.	*Example*: $3x+4y=13$, $\frac{1}{3}x-2y=-1$.
	Evaluating exponential expressions with integer values.	*Examples*: a^b, $b\in\mathbb{Z}$; $2^{-4}=\frac{1}{16}$; $(-2)^4=16$.
	Use of inequalities $<$, $\leq$, $>$, $\geq$. Intervals on the real number line.	*Example*: $2<x\leq5$, $x\in\mathbb{R}$.
	Solving linear inequalities.	*Example*: $2x+5<7-x$.
	Familiarity with commonly accepted world currencies.	*Examples*: Swiss franc (CHF); United States dollar (USD); British pound sterling (GBP); euro (EUR); Japanese yen (JPY); Australian dollar (AUD).

	Content	Further guidance
2.0	The collection of data and its representation in bar charts, pie charts and pictograms.	
5.0	Basic geometric concepts: point, line, plane, angle. Simple two-dimensional shapes and their properties, including perimeters and areas of circles, triangles, quadrilaterals and compound shapes. SI units for length and area. Pythagoras' theorem. Coordinates in two dimensions. Midpoints, distance between points.	

Syllabus content

Topic 1—Number and algebra 20 hours

The aims of this topic are to introduce some basic elements and concepts of mathematics, and to link these to financial and other applications.

	Content	Further guidance	Links
1.1	Natural numbers, $\mathbb{N}$; integers, $\mathbb{Z}$; rational numbers, $\mathbb{Q}$; and real numbers, $\mathbb{R}$. **Not required:** proof of irrationality, for example, of $\sqrt{2}$.	Link with domain and range 6.1.	**Int:** Historical development of number system. Awareness that our modern numerals are developed from the Arabic notation. **TOK:** Do mathematical symbols have sense in the same way that words have sense? Is zero different? Are these numbers created or discovered? Do these numbers exist?
1.2	Approximation: decimal places, significant figures. Percentage errors. Estimation.	Students should be aware of the errors that can result from premature rounding. Students should be able to recognize whether the results of calculations are reasonable, including reasonable values of, for example, lengths, angles and areas. For example, lengths cannot be negative.	**Appl:** Currency approximations to nearest whole number, eg peso, yen. Currency approximations to nearest cent/penny, eg euro, dollar, pound. **Appl:** Physics 1.1 (range of magnitudes). **Appl:** Meteorology, alternative rounding methods. **Appl:** Biology 2.1.5 (microscopic measurement). **TOK:** Appreciation of the differences of scale in number, and of the way numbers are used that are well beyond our everyday experience.

	Content	Further guidance	Links
1.3	Expressing numbers in the form $a \times 10^k$, where $1 \le a < 10$ and k is an integer. Operations with numbers in this form.	Students should be able to use scientific mode on the GDC. Calculator notation is *not* acceptable. For example, 5.2E3 is *not* acceptable.	**Appl:** Very large and very small numbers, eg astronomical distances, sub-atomic particles; Physics 1.1; global financial figures. **Appl:** Chemistry 1.1 (Avogadro's number). **Appl:** Physics 1.2 (scientific notation). **Appl:** Chemistry and biology (scientific notation). **Appl:** Earth science (earthquake measurement scale).
1.4	SI (*Système International*) and other basic units of measurement: for example, kilogram (kg), metre (m), second (s), litre (l), metre per second ($m\ s^{-1}$), Celsius scale.	Students should be able to convert between different units. Link with the form of the notation in 1.3, for example, $5\text{km} = 5 \times 10^6 \text{mm}$.	**Appl:** Speed, acceleration, force; Physics 2.1, Physics 2.2; concentration of a solution; Chemistry 1.5. **Int:** SI notation. **TOK:** Does the use of SI notation help us to think of mathematics as a "universal language"? **TOK:** What is measurable? How can one measure mathematical ability?
1.5	Currency conversions.	Students should be able to perform currency transactions involving commission.	**Appl:** Economics 3.2 (exchange rates). **Aim 8:** The ethical implications of trading in currency and its effect on different national communities. **Int:** The effect of fluctuations in currency rates on international trade.

Syllabus content

	Content	Further guidance	Links
1.6	Use of a GDC to solve • pairs of linear equations in two variables • quadratic equations.	In examinations, no specific method of solution will be required. Standard terminology, such as zeros or roots, should be taught. Link with quadratic models in 6.3.	**TOK:** Equations with no solutions. Awareness that when mathematicians talk about "imaginary" or "real" solutions they are using precise technical terms that do not have the same meaning as the everyday terms.
1.7	Arithmetic sequences and series, and their applications. Use of the formulae for the *n*th term and the sum of the first *n* terms of the sequence.	Students may use a GDC for calculations, but they will be expected to identify the first term and the common difference.	**TOK:** Informal and formal reasoning in mathematics. How does mathematical proof differ from good reasoning in everyday life? Is mathematical reasoning different from scientific reasoning? **TOK:** Beauty and elegance in mathematics. Fibonacci numbers and connections with the Golden ratio.
1.8	Geometric sequences and series. Use of the formulae for the *n*th term and the sum of the first *n* terms of the sequence. **Not required:** formal proofs of formulae. **Not required:** use of logarithms to find *n*, given the sum of the first *n* terms; sums to infinity.	Students may use a GDC for calculations, but they will be expected to identify the first term and the common ratio.	

	Content	Further guidance	Links
1.9	Financial applications of geometric sequences and series: • compound interest • annual depreciation. **Not required:** use of logarithms.	Use of the GDC is expected, including built-in financial packages. The concept of simple interest may be used as an introduction to compound interest but will not be examined. In examinations, questions that ask students to derive the formula will not be set. Compound interest can be calculated yearly, half-yearly, quarterly or monthly. Link with exponential models 6.4.	**Appl:** Economics 3.2 (exchange rates). **Aim 8:** Ethical perceptions of borrowing and lending money. **Int:** Do all societies view investment and interest in the same way?

Topic 2—Descriptive statistics 12 hours

The aim of this topic is to develop techniques to describe and interpret sets of data, in preparation for further statistical applications.

	Content	Further guidance	Links
2.1	Classification of data as discrete or continuous.	Students should understand the concept of population and of representative and random sampling. Sampling will not be examined but can be used in internal assessment.	**Appl:** Psychology 3 (research methodology). **Appl:** Biology 1 (statistical analysis). **TOK:** Validity of data and introduction of bias.
2.2	Simple discrete data: frequency tables.		
2.3	Grouped discrete or continuous data: frequency tables; mid-interval values; upper and lower boundaries. Frequency histograms.	In examinations, frequency histograms will have equal class intervals.	**Appl:** Geography (geographical analyses).
2.4	Cumulative frequency tables for grouped discrete data and for grouped continuous data; cumulative frequency curves, median and quartiles. Box-and-whisker diagram. **Not required:** treatment of outliers.	Use of GDC to produce histograms and box-and-whisker diagrams.	
2.5	Measures of central tendency. For simple discrete data: mean; median; mode. For grouped discrete and continuous data: estimate of a mean; modal class.	Students should use mid-interval values to estimate the mean of grouped data. In examinations, questions using Σ notation will not be set.	**Aim 8:** The ethical implications of using statistics to mislead.

	Content	Further guidance	Links
2.6	Measures of dispersion: range, interquartile range, standard deviation.	Students should use mid-interval values to estimate the standard deviation of grouped data. In examinations: • students are expected to use a GDC to calculate standard deviations • the data set will be treated as the population. Students should be aware that the IB notation may differ from the notation on their GDC. Use of computer spreadsheet software is encouraged in the treatment of this topic.	**Int:** The benefits of sharing and analysing data from different countries. **TOK:** Is standard deviation a mathematical discovery or a creation of the human mind?

Topic 3—Logic, sets and probability 20 hours

The aims of this topic are to introduce the principles of logic, to use set theory to introduce probability, and to determine the likelihood of random events using a variety of techniques.

	Content	Further guidance	Links
3.1	Basic concepts of symbolic logic: definition of a proposition; symbolic notation of propositions.		
3.2	Compound statements: implication, $\Rightarrow$; equivalence, $\Leftrightarrow$; negation, $\neg$; conjunction, $\wedge$; disjunction, $\vee$; exclusive disjunction, $\underline{\vee}$. Translation between verbal statements and symbolic form.		
3.3	Truth tables: concepts of logical contradiction and tautology.	A maximum of three propositions will be used in truth tables. Truth tables can be used to illustrate the associative and distributive properties of connectives, and for variations of implication and equivalence statements, for example, $\neg q \Rightarrow \neg p$.	
3.4	Converse, inverse, contrapositive. Logical equivalence. Testing the validity of simple arguments through the use of truth tables.	The topic may be extended to include syllogisms. In examinations these will not be tested.	**Appl:** Use of arguments in developing a logical essay structure. **Appl:** Computer programming; digital circuits; Physics HL 14.1; Physics SL C1. **TOK:** Inductive and deductive logic, fallacies.

	Content	Further guidance	Links
3.5	Basic concepts of set theory: elements $x \in A$, subsets $A \subset B$; intersection $A \cap B$; union $A \cup B$; complement A'. Venn diagrams and simple applications. **Not required:** knowledge of de Morgan's laws.	In examinations, the universal set U will include no more than three subsets. The empty set is denoted by $\varnothing$.	
3.6	Sample space; event A; complementary event, A'. Probability of an event. Probability of a complementary event. Expected value.	Probability may be introduced and taught in a practical way using coins, dice, playing cards and other examples to demonstrate random behaviour. In examinations, questions involving playing cards will not be set.	**Appl:** Actuarial studies, probability of life spans and their effect on insurance. **Appl:** Government planning based on projected figures. **TOK:** Theoretical and experimental probability.
3.7	Probability of combined events, mutually exclusive events, independent events. Use of tree diagrams, Venn diagrams, sample space diagrams and tables of outcomes. Probability using "with replacement" and "without replacement". Conditional probability.	Students should be encouraged to use the most appropriate method in solving individual questions. Probability questions will be placed in context and will make use of diagrammatic representations. In examinations, questions requiring the exclusive use of the formula in section 3.7 of the formula booklet will not be set.	**Appl:** Biology 4.3 (theoretical genetics); Biology 4.3.2 (Punnett squares). **Appl:** Physics HL13.1 (determining the position of an electron); Physics SL B1. **Aim 8:** The ethics of gambling. **TOK:** The perception of risk, in business, in medicine and safety in travel.

Topic 4—Statistical applications 17 hours

The aims of this topic are to develop techniques in inferential statistics in order to analyse sets of data, draw conclusions and interpret these.

	Content	Further guidance	Links
4.1	The normal distribution. The concept of a random variable; of the parameters μ and σ; of the bell shape; the symmetry about $x = \mu$.	Students should be aware that approximately 68% of the data lies between $\mu \pm \sigma$, 95% lies between $\mu \pm 2\sigma$ and 99% lies between $\mu \pm 3\sigma$.	**Appl:** Examples of measurements, ranging from psychological to physical phenomena, that can be approximated, to varying degrees, by the normal distribution. **Appl:** Biology 1 (statistical analysis). **Appl:** Physics 3.2 (kinetic molecular theory).
	Diagrammatic representation.	Use of sketches of normal curves and shading when using the GDC is expected.	
	Normal probability calculations.	Students will be expected to use the GDC when calculating probabilities and inverse normal.	
	Expected value.		
	Inverse normal calculations.	In examinations, inverse normal questions will not involve finding the mean or standard deviation.	
	Not required: Transformation of any normal variable to the standardized normal.	Transformation of any normal variable to the standardized normal variable, z, may be appropriate in internal assessment. In examinations, questions requiring the use of z scores will not be set.	

	Content	Further guidance	Links
4.2	Bivariate data: the concept of correlation.	Students should be able to make the distinction between correlation and causation.	**Appl:** Biology; Physics; Chemistry; Social sciences. **TOK:** Does correlation imply causation?
	Scatter diagrams; line of best fit, by eye, passing through the mean point.		
	Pearson's product–moment correlation coefficient, r.	Hand calculations of r may enhance understanding. In examinations, students will be expected to use a GDC to calculate r.	
	Interpretation of positive, zero and negative, strong or weak correlations.		
4.3	The regression line for y on x.	Hand calculations of the regression line may enhance understanding. In examinations, students will be expected to use a GDC to find the regression line.	**Appl:** Chemistry 11.3 (graphical techniques). **TOK:** Can we reliably use the equation of the regression line to make predictions?
	Use of the regression line for prediction purposes.	Students should be aware of the dangers of extrapolation.	

	Content	Further guidance	Links
4.4	The χ^2 test for independence: formulation of null and alternative hypotheses; significance levels; contingency tables; expected frequencies; degrees of freedom; p-values.	In examinations: • the maximum number of rows or columns in a contingency table will be 4 • the degrees of freedom will always be greater than one • the χ^2 critical value will always be given • only questions on upper tail tests with commonly used significance levels (1%, 5%, 10%) will be set. Calculation of expected frequencies by hand is required. Hand calculations of χ^2 may enhance understanding. In examinations students will be expected to use the GDC to calculate the χ^2 statistic. If using χ^2 tests in internal assessment, students should be aware of the limitations of the test for small expected frequencies; **expected frequencie**s must be greater than 5. If the degree of freedom is 1, then Yates's continuity correction should be applied.	**Appl:** Biology (internal assessment); Psychology; Geography. **TOK:** Scientific method.

Topic 5—Geometry and trigonometry

18 hours

The aims of this topic are to develop the ability to draw clear diagrams in two dimensions, and to apply appropriate geometric and trigonometric techniques to problem-solving in two and three dimensions.

	Content	Further guidance	Links
5.1	Equation of a line in two dimensions: the forms $y = mx + c$ and $ax + by + d = 0$.	Link with linear functions in 6.2.	**Appl:** Gradients of mountain roads, eg Canadian Highway. Gradients of access ramps. **Appl:** Economics 1.2 (elasticity). **TOK:** Descartes showed that geometric problems can be solved algebraically and vice versa. What does this tell us about mathematical representation and mathematical knowledge?
	Gradient; intercepts.		
	Points of intersection of lines.	Link with solutions of pairs of linear equations in 1.6.	
	Lines with gradients, m_1 and m_2. Parallel lines $m_1 = m_2$. Perpendicular lines, $m_1 \times m_2 = -1$.		
5.2	Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles. Angles of elevation and depression.	Problems may incorporate Pythagoras' theorem. In examinations, questions will only be set in degrees.	**Appl:** Triangulation, map-making, finding practical measurements using trigonometry. **Int:** Diagrams of Pythagoras' theorem occur in early Chinese and Indian manuscripts. The earliest references to trigonometry are in Indian mathematics.

	Content	Further guidance	Links
5.3	Use of the sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.	In all areas of this topic, students should be encouraged to sketch well-labelled diagrams to support their solutions. The ambiguous case could be taught, but will not be examined. In examinations, questions will only be set in degrees.	**Appl:** Vectors; Physics 1.3; bearings.
	Use of the cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$; $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.		**TOK:** Use the fact that the cosine rule is one possible generalization of Pythagoras' theorem to explore the concept of "generality".
	Use of area of a triangle $= \frac{1}{2}ab\sin C$. Construction of labelled diagrams from verbal statements.		

	Content	Further guidance	Links
5.4	Geometry of three-dimensional solids: cuboid; right prism; right pyramid; right cone; cylinder; sphere; hemisphere; and combinations of these solids. The distance between two points; eg between two vertices or vertices with midpoints or midpoints with midpoints. The size of an angle between two lines or between a line and a plane. **Not required:** angle between two planes.	In examinations, only right-angled trigonometry questions will be set in reference to three-dimensional shapes.	**TOK:** What is an axiomatic system? Do the angles in a triangle always add to 180°? Non-Euclidean geometry, such as Riemann's. Flight maps of airlines. **Appl:** Architecture and design.
5.5	Volume and surface areas of the three-dimensional solids defined in 5.4.		

Topic 6—Mathematical models 20 hours

The aim of this topic is to develop understanding of some mathematical functions that can be used to model practical situations. Extensive use of a GDC is to be encouraged in this topic.

	Content	Further guidance	Links
6.1	Concept of a function, domain, range and graph. Function notation, eg $f(x)$, $v(t)$, $C(n)$. Concept of a function as a mathematical model.	In examinations: • the domain is the set of all real numbers unless otherwise stated • mapping notation $f: x \mapsto y$ will not be used.	**TOK:** Why can we use mathematics to describe the world and make predictions? Is it because we discover the mathematical basis of the world or because we impose our own mathematical structures onto the world? The relationship between real-world problems and mathematical models.
6.2	Linear models. Linear functions and their graphs, $f(x) = mx + c$.	Link with equation of a line in 5.1.	**Appl:** Conversion graphs, eg temperature or currency conversion; Physics 3.1; Economics 3.2.
6.3	Quadratic models. Quadratic functions and their graphs (parabolas): $f(x) = ax^2 + bx + c$; $a \neq 0$ Properties of a parabola: symmetry; vertex; intercepts on the x-axis and y-axis. Equation of the axis of symmetry, $x = -\frac{b}{2a}$.	Link with the quadratic equations in 1.6. Functions with zero, one or two real roots are included. The form of the equation of the axis of symmetry may initially be found by investigation. Properties should be illustrated with a GDC or graphical software.	**Appl:** Cost functions; projectile motion; Physics 9.1; area functions.

	Content	Further guidance	Links
6.4	Exponential models. Exponential functions and their graphs: $f(x) = ka^x + c;\ a \in \mathbb{Q}^+,\ a \neq 1,\ k \neq 0$. $f(x) = ka^{-x} + c;\ a \in \mathbb{Q}^+,\ a \neq 1,\ k \neq 0$. Concept and equation of a horizontal asymptote.	In examinations, students will be expected to use graphical methods, including GDCs, to solve problems.	**Appl:** Biology 5.3 (populations). **Appl:** Biology 5.3.2 (population growth); Physics 13.2 (radioactive decay); Physics I2 (X-ray attenuation); cooling of a liquid; spread of a virus; depreciation.
6.5	Models using functions of the form $f(x) = ax^m + bx^n + \ldots;\ m, n \in \mathbb{Z}$. Functions of this type and their graphs. The y-axis as a vertical asymptote.	In examinations, students will be expected to use graphical methods, including GDCs, to solve problems. *Examples*: $f(x) = 3x^4 - 5x + 3$, $g(x) = 3x^2 - \frac{4}{x}$.	
6.6	Drawing accurate graphs. Creating a sketch from information given. Transferring a graph from GDC to paper. Reading, interpreting and making predictions using graphs. Included all the functions above and additions and subtractions.	Students should be aware of the difference between the command terms “draw” and “sketch”. All graphs should be labelled and have some indication of scale. *Examples*: $f(x) = x^3 + 5 - \frac{2}{x}$, $g(x) = 3^{-x} + x$.	**TOK:** Does a graph without labels or indication of scale have meaning?

	Content	Further guidance	Links
6.7	Use of a GDC to solve equations involving combinations of the functions above.	*Examples*: $x + 2 = 2x^3 + 3x - 1$, $5x = 3^x$. Other functions can be used for modelling in internal assessment but will not be set on examination papers.	

Topic 7—Introduction to differential calculus

18 hours

The aim of this topic is to introduce the concept of the derivative of a function and to apply it to optimization and other problems.

	Content	Further guidance	Links
7.1	Concept of the derivative as a rate of change. Tangent to a curve. **Not required:** formal treatment of limits.	Teachers are encouraged to introduce differentiation through a graphical approach, rather than a formal treatment. Emphasis is placed on interpretation of the concept in different contexts. In examinations, questions on differentiation from first principles will not be set.	**Appl:** Rates of change in economics, kinematics and medicine. **Aim 8:** Plagiarism and acknowledgment of sources, eg the conflict between Newton and Leibnitz, who approached the development of calculus from different directions **TOK:** Is intuition a valid way of knowing in maths? How is it possible to reach the same conclusion from different research paths?
7.2	The principle that $f(x)=ax^n \Rightarrow f'(x)=anx^{n-1}$. The derivative of functions of the form $f(x)=ax^n+bx^{n-1}+\ldots$, where all exponents are integers.	Students should be familiar with the alternative notation for derivatives $\frac{dy}{dx}$ or $\frac{dV}{dr}$. In examinations, knowledge of the second derivative will not be assumed.	

	Content	Further guidance	Links
7.3	Gradients of curves for given values of x. Values of x where $f'(x)$ is given. Equation of the tangent at a given point. Equation of the line perpendicular to the tangent at a given point (normal).	The use of technology to find the gradient at a point is also encouraged. The use of technology to draw tangent and normal lines is also encouraged. Links with perpendicular lines in 5.1.	
7.4	Increasing and decreasing functions. Graphical interpretation of $f'(x)>0$, $f'(x)=0$ and $f'(x)<0$.		
7.5	Values of x where the gradient of a curve is zero. Solution of $f'(x)=0$. Stationary points. Local maximum and minimum points.	The use of technology to display $f(x)$ and $f'(x)$, and find the solutions of $f'(x)=0$ is also encouraged. Awareness that a local maximum/minimum will not necessarily be the greatest/least value of the function in the given domain. Awareness of points of inflexion with zero gradient is to be encouraged, but will not be examined.	
7.6	Optimization problems.	*Examples*: Maximizing profit, minimizing cost, maximizing volume for given surface area. In examinations, questions on kinematics will not be set.	**Appl:** Efficient use of material in packaging. **Appl:** Physics 2.1 (kinematics).